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## Beyond Snel's law: Refraction of a nano-beam of light

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### ABSTRACT

The refraction of a localized narrow beam is significantly different from that of a plane wave. As the beam width decreases to be in the order of the wavelength, the refraction behavior deviates noticeably from Snel's law, and when the width of a light beam is smaller than about one fifth of the wavelength of the incident light, finite-difference time-domain simulations demonstrate that refraction becomes negligible. That is, the narrow light beam retains its propagation direction even after entering another medium at an oblique angle. The result reveals novel features of nano-beams and may have applications in precise biomedical measurement or micro optical device.

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### 1. Introduction

Light refraction is one of the most commonly observed optical phenomena. When light wave is transmitted from one optical medium into another at an oblique angle, its direction of propagation changes. If the light is a wide plane wave incident on a relatively large and flat interface of two different isotropic media, the change of wave direction is well described by Snel's law. Snel's Law of refraction is one of the oldest fundamental laws of optics [1] used in virtually all engineering applications where refraction exists, such as in designing lenses and filters, fiber optics for telecommunications, and increasing detector efficiencies. It has recently been discovered that light interacting with matter having nano-scale structures behaves differently than what we have grown accustomed to with micro-scale structures [2]. It is expected to find analogous, unexpected results without the nano-structures by simply confining light to narrow regions.

With the advent of microscopic lasers that can reach the diffraction limit [3–7], a new area of optical physics has

emerged. Recently, Nakayama et al. [8] demonstrated a green light laser beam with a wavelength of  $531 \pm 1.8$  nm from a nanowire. Although not specifically emphasized, the width of the beam from the nanowire is estimated to be not larger than 50 nm based on the results in Fig. 3 of Nakayama et al. [8]. Also, Oulton et al. [9] reported an experimental demonstration of nanometer-scale plasmonic lasers, generating optical modes 100 times smaller than the diffraction limit. Although the field amplitude and phase distribution of these thin beams are not currently well known, these reports already have declared that the conventional diffraction limit that prevents the width of light beam from being smaller than one half wavelength has been broken by their novel laboratory experimental results. Therefore, a practical method to establish a laser beam whose radius is in the order of one tenth of a wavelength already exists. Since these highly localized lasers are beyond the diffraction limit, the refraction of these narrow beams may be significantly different from that of a conventional beam, and the most fundamental law of refraction, Snel's Law, may not be obeyed.

### 2. Method

In this study, the refraction of narrow light beams is calculated by the three-dimensional (3D) finite-difference

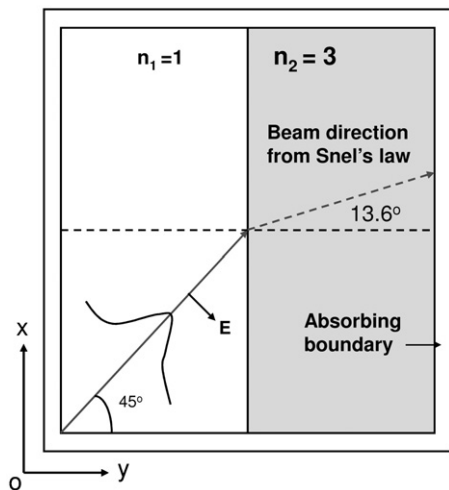
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time-domain (FDTD) method [10–14]. The FDTD technique is used to calculate electromagnetic fields in the time domain by directly solving the finite-differenced Maxwell's equations. The electromagnetic properties of the media are specified by assigning the permittivity, permeability, and conductivity at each grid point of the FDTD mesh. A time-stepping iteration is used to simulate the field variation with time. Since the phase and amplitude distribution within the reported thin lasers are not known currently, the incident light beam is assumed to be simply a continuous wave with a Gaussian profile, i.e. its field amplitude is modulated by a Gaussian distribution factor circularly symmetric to the central axis of the beam:

$$A = \exp \left[ -\left( \frac{d}{\sigma} \right)^2 \right], \quad (1)$$

where  $d$  is the radial distance from the central axis of the beam, and  $\sigma$  is a constant radius at which the field amplitude drops to  $1/e$  of its central value. The accuracy of the FDTD results depends on the spatial cell size. To ensure a highly accurate numerical result, the spatial cell size of the FDTD is set to be  $1/60$  of the incident wavelength and the time step is  $1/2$  of the time that the wave can travel through a single cell in free space. The computational domain is illustrated in Fig. 1, which is the xoy cross-section of a 3D space and is truncated by an absorbing boundary [14–16]. In this illustration, the beam propagates from free space to a material with a refractive index of 3. The central axis of the beam passes the central point of the material interface at an incident angle of  $45^\circ$ . The beam is a polarized wave with the electric field vector parallel to the xoy plane and perpendicular to the propagation direction.

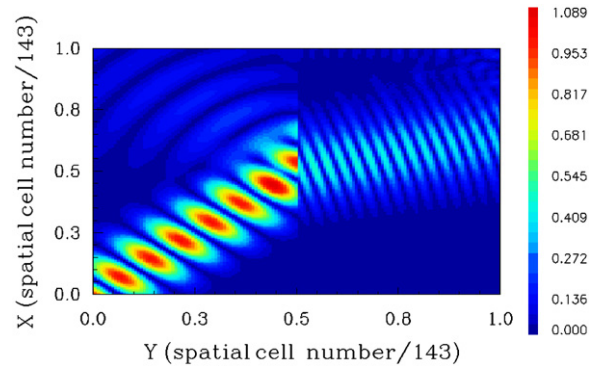


**Fig. 1.** Illustration of a continuous wave beam incident on a smooth material surface at  $45^\circ$ . The refractive index of the material space is 3. The xoy plane is a cross-section of the 3D FDTD computational domain which has  $90 \times 90 \times 90$  FDTD cubic cells. The absorbing boundary is 6 cubic cells in depth.

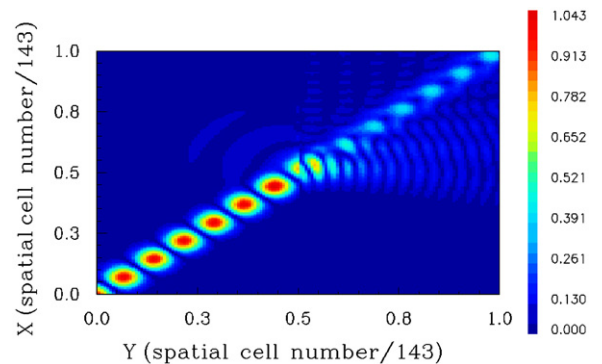
### 3. Results

Fig. 2 shows the electric field intensity ( $|E|^2$ ) at the 900th time step of the FDTD simulation. The electric field intensity is normalized by the incident electric field intensity along the central axis of the incident beam. The beam width ( $2\sigma$ ) is 1 incident wavelength. We can see the beam in Fig. 2 has a strong refraction within the material, which is very close to what is predicted from Snell's law even though the beam is quite narrow. However, when the beam width is set as  $2/5$  of the incident wavelength, as shown in Fig. 3, the beam diverges into two branches, one beam traveling along the original incident direction and the other beam traveling in a different direction. Most interestingly, when the beam width is set to be  $1/5$  of the incident wavelength, as shown in Fig. 4, the surface no longer refracts the beam, and the wave continues to propagate in its original incidence direction. This is definitely a case beyond Snell's law.

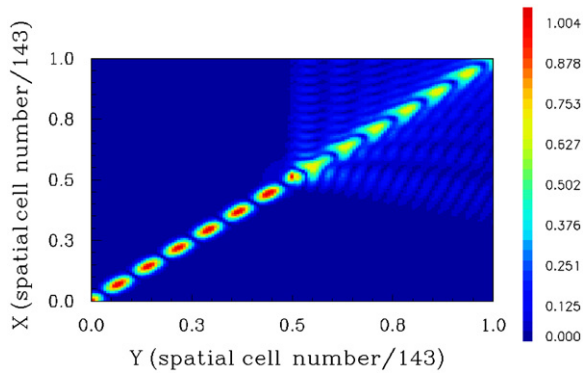
That Snell's law is not obeyed for very narrow beams should not be a surprise when we consider that refraction can be thought of as an interference phenomenon in which destructive interference of all the wavelets along the infinite surface removes all but the light that travels in compliance of Snell's law. In the case of a nano-beam, this destructive interference does not exist, so the beam retains its original direction of propagation.



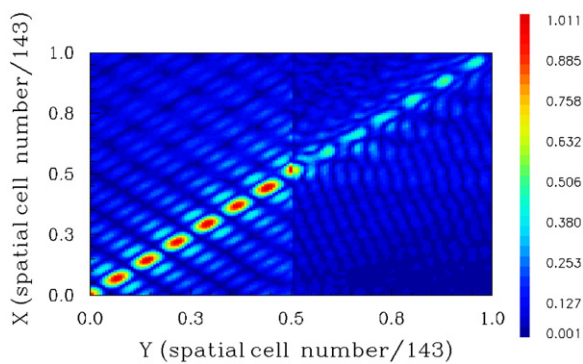
**Fig. 2.** The electric field intensity ( $|E|^2$ ) at the 900th time step of the FDTD simulation for a system illustrated in Fig. 1. The beam width ( $2\sigma$ ) is 1 incident wavelength.



**Fig. 3.** Same as in Fig. 2, but for a beam width ( $2\sigma$ ) of  $2/5$  incident wavelength.



**Fig. 4.** Same as in Fig. 2, but for a beam width ( $2\sigma$ ) of  $1/5$  incident wavelength.



**Fig. 5.** Same as in Fig. 2, but for a Bessel-shaped beam with a central spot of  $\sim 1/5$  incident wavelength.

Since the phase and amplitude distribution of the thin lasers are not reported in [8,9], we assumed the incident light beam to be simply a continuous wave with a Gaussian profile in this study. This may not be true for the nature of the reported nanometer lasers. However, we believe that the non-refraction phenomena of the thin lasers are mainly determined by the beam size. Whether the assumed Gaussian-profiled plane wave beam is practical or not does not affect the conclusion in this report. To strengthen this idea, we tested an incidence with radial intensity distribution as a Bessel-shaped beam [17], which is a strong narrow central beam circled by weak concentric rings. The Bessel-shaped beam can propagate without being subjected to diffractive spreading. Fig. 5 shows the electric field intensity ( $|E|^2$ ) at the 900th time step of the FDTD simulation with the incidence of Bessel-shaped beam. The central peak of the beam is set to be  $\sim 1/5$  of the incident wavelength. We can see that the narrow central mode enters the material space without refraction either.

#### 4. Conclusion

The refraction of a localized narrow beam is significantly different from that of a plane wave. For reported nanometer

beams as in [8,9], geometric optics laws including Snell's Law should be violated. As the width of a light beam is smaller than about one fifth of the wavelength of the incident light, finite-difference time-domain simulations demonstrate that refraction becomes negligible. The result reveals novel features of nano-beams. However, in an asymptotical case of an extreme narrow beam, the incidence approaches a light point source situated on the interface. The total electromagnetic field should be quite different from the patterns shown in Figs. 2–5. In such a case, the concepts of the beam propagation direction inside a medium and Snell's Law become invalid.

As progress in nanotechnologies continues, directing and focusing electromagnetic energy into small regions will become commonplace. We show that, like many aspects of nanotechnology, accurate modeling is essential as our tools and experience gained with macro-scale or even micro-scale technologies are not adequate. In this case, we demonstrate that the most fundamental law of refraction breaks down when the beam width is brought to the nanometer scale.

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